# HOSSAM GHANEM

## (3) 2.3 Techniques for finding limits (B)

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| $(x-a)(x+a)=x^2-a^2$ | $(\sqrt{x}+3)(\sqrt{x}-3)=x-9$<br>$(\sqrt{x+5}-\sqrt{2x-1})(\sqrt{x+5}+\sqrt{2x-1})=(x+5)-(2x-1)$<br>$\left(\sqrt{3x+1}-(2x-5)\right)\left(\sqrt{3x+1}+(2x-5)\right)=(3x+1)-(2x-5)^2$ |

| Example 1<br>53 July 18, 2009 A | Find the limit , if it exists $\lim_{x\to1}\dfrac{\sqrt{x+8}-3}{x^2+8x-9}$ |
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## Solution

$$\lim_{x\to1}\frac{\sqrt{x+8}-3}{x^2+8x-9}=\frac{3-3}{1+8-9}=\frac{0}{0}$$

$$\lim_{x\to1}\frac{\sqrt{x+8}-3}{x^2+8x-9}=\lim_{x\to1}\frac{(\sqrt{x+8}-3)(\sqrt{x+8}+3)}{(x^2+8x-9)(\sqrt{x+8}+3)}=\lim_{x\to3}\frac{x+8-9}{(x+9)(x-1)(\sqrt{x+8}+3)}$$

$$=\lim_{x\to1}\frac{(x-1)}{(x+9)(x-1)(\sqrt{x+8}+3)}=\lim_{x\to1}\frac{1}{(x+9)(\sqrt{x+8}+3)}=\frac{1}{10(3+3)}=\frac{1}{60}$$

| Example 2<br>43 June 28, 2008 | Evaluate the following limit $\lim_{x\to1}\dfrac{x-1}{\sqrt{2x}-\sqrt{3-x}}$ |
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## Solution

$$\lim_{x\to1}\frac{x-1}{\sqrt{2x}-\sqrt{3-x}}=\frac{1-1}{\sqrt2-\sqrt2}=\frac{0}{0}$$

$$\lim_{x\to1}\frac{x-1}{\sqrt{2x}-\sqrt{3-x}}=\lim_{x\to1}\frac{(x-1)(\sqrt{2x}+\sqrt{3-x})}{(\sqrt{2x}-\sqrt{3-x})(\sqrt{2x}+\sqrt{3-x})}=\lim_{x\to1}\frac{(x-1)(\sqrt{2x}+\sqrt{3-x})}{2x-(3-x)}$$

$$=\lim_{x\to1}\frac{(x-1)(\sqrt{2x}+\sqrt{3-x})}{2x-3+x}=\lim_{x\to1}\frac{(x-1)(\sqrt{2x}+\sqrt{3-x})}{3x-3}=\lim_{x\to1}\frac{(x-1)(\sqrt{2x}+\sqrt{3-x})}{3(x-1)}$$

$$=\lim_{x\to1}\frac{(\sqrt{2x}+\sqrt{3-x})}{3}=\frac{\sqrt2+\sqrt2}{3}=\frac{2\sqrt2}{3}$$

| Example 3<br>13 November 13, 1995 | Find the limit , if it exists $\lim_{x\to1}\dfrac{x+\sqrt{8x+1}-4}{x-1}$ |
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## Solution

$$\lim_{x\to1}\frac{x+\sqrt{8x+1}-4}{x-1}=\frac{1+3-4}{1-1}=\frac{0}{0}$$

$$\lim_{x\to1}\frac{x+\sqrt{8x+1}-4}{x-1}=\lim_{x\to1}\frac{(x-4)+\sqrt{8x+1}}{x-1}=\lim_{x\to1}\frac{[(x-4)+\sqrt{8x+1}][(x-4)-\sqrt{8x+1}]}{(x-1)[(x-4)-\sqrt{8x+1}]}$$

$$=\lim_{x\to1}\frac{(x^2-4)^2-(8x+1)}{(x-1)[(x-4)-\sqrt{8x+1}]}=\lim_{x\to1}\frac{x^2-8x+16-8x-1}{(x-1)[(x-4)-\sqrt{8x+1}]}$$

$$=\lim_{x\to1}\frac{x^2-16x+15}{(x-1)[(x-4)-\sqrt{8x+1}]}=\lim_{x\to1}\frac{(x-1)(x-15)}{(x-1)[(x-4)-\sqrt{8x+1}]}$$

$$=\lim_{x\to1}\frac{(x-15)}{(x-4)-\sqrt{8x+1}}=\frac{1-15}{1-4-3}=\frac{-14}{-6}=\frac{7}{3}$$

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| **Example 4**<br>28 October 24, 1999 | Evaluate the following limit $\lim_{x\to1}\frac{\sqrt[3]{x+7}-2}{x-1}$ |

## Solution

$$\lim_{x\to1}\frac{\sqrt[3]{x+7}-2}{x-1}=\frac{\sqrt[3]{8}-2}{1-1}=\frac{2-2}{1-1}=\frac{0}{0}$$

$$\lim_{x\to1}\frac{\sqrt[3]{x+7}-2}{x-1}=\lim_{x\to1}\frac{\sqrt[3]{x+7}-2}{(x+7)-8}=\lim_{x\to1}\frac{\sqrt[3]{x+7}-2}{\left(\sqrt[3]{x+7}-2\right)\left[\left(\sqrt[3]{x+7}\right)^2+2\sqrt[3]{x+7}+4\right]}$$

$$=\lim_{x\to1}\frac{1}{\left(\sqrt[3]{x+7}\right)^2+2\sqrt[3]{x+7}+4}=\frac{1}{4+4+4}=\frac{1}{12}$$

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| **Example 5**<br>5 April 8, 1993 | Let the function $f$ be given by $f(x)=\begin{cases}\frac{\sqrt{2x+3}-\sqrt{x+2}}{x+1} & \text{if} \quad x>-1\\ \frac{x^2-1}{x^2-x-2} & \text{if} \quad x<-1\end{cases}$<br>Show that $\lim_{x\to-1}f(x)$ does not exist |

## Solution

$$\lim_{x\to-1^-}f(x)=\lim_{x\to-1^-}\frac{x^2-1}{x^2-x-2}=\frac{1-1}{1+1-2}=\frac{0}{0}$$

$$\lim_{x\to-1^-}f(x)=\lim_{x\to-1^-}\frac{(x-1)(x+1)}{(x-2)(x+1)}=\lim_{x\to-1^-}\frac{(x-1)}{(x-2)}=\frac{-2}{-3}=\frac{2}{3}$$

$$\lim_{x\to-1^+}f(x)=\lim_{x\to-1^+}\frac{\sqrt{2x+3}-\sqrt{x+2}}{x+1}=\frac{1-1}{-1+1}=\frac{0}{0}$$

$$=\lim_{x\to-1^+}\frac{\left(\sqrt{2x+3}-\sqrt{x+2}\right)\left(\sqrt{2x+3}+\sqrt{x+2}\right)}{(x+1)\left(\sqrt{2x+3}+\sqrt{x+2}\right)}=\lim_{x\to-1^+}\frac{2x+3-(x+2)}{(x+1)\left(\sqrt{2x+3}+\sqrt{x+2}\right)}$$

$$=\lim_{x\to-1^+}\frac{2x+3-x-2}{(x+1)\left(\sqrt{2x+3}+\sqrt{x+2}\right)}=\lim_{x\to-1^+}\frac{(x+1)}{(x+1)\left(\sqrt{2x+3}+\sqrt{x+2}\right)}$$

$$=\lim_{x\to-1^+}\frac{1}{\left(\sqrt{2x+3}+\sqrt{x+2}\right)}=\frac{1}{1+1}=\frac{1}{2}$$

$\therefore\ \lim_{x\to-1^-}f(x)\neq\lim_{x\to-1^+}f(x)$

$\therefore\ \lim_{x\to-1}f(x)\ \ D.N.E$

| Example 6<br>33 January 20, 2009 | Evaluate the following limit $\lim_{x\to2}\left(\frac{1}{x-2}-\frac{4}{x^2-4}\right)$ |
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## Solution

$$\lim_{x\to2}\left(\frac{1}{x-2}-\frac{4}{x^2-4}\right)=\infty-\infty$$

$$\lim_{x\to2}\left(\frac{1}{x-2}-\frac{4}{x^2-4}\right)=\lim_{x\to2}\frac{1}{(x-2)}-\frac{4}{(x-2)(x+2)}$$

$$=\lim_{x\to2}\frac{x+2-4}{(x-2)(x+2)}=\lim_{x\to2}\frac{x-2}{(x-2)(x+2)}=\lim_{x\to2}\frac{1}{(x+2)}=\frac{1}{4}$$

**Example 7** 34 June 21, 2009

1. Let $f(x)$ and $g(x)$ be two functions such that $\lim_{x\to1}(f(x)+g(x))=2$

If $f(x)=3x+\frac{5}{x}$ and $\lim_{x\to1}g(x)$ exists, find $\lim_{x\to1}g(x)$ (4 pts.)

## Solution

$$\lim_{x\to1}f(x)=\lim_{x\to1}3x+\frac{5}{x}=3+5=8$$

$$\lim_{x\to1}(f(x)+g(x))=2$$

$$\lim_{x\to1}f(x)+\lim_{x\to1}g(x)=2$$

$$8+\lim_{x\to1}g(x)=2$$

$$\lim_{x\to1}g(x)=-6$$

| Example 8<br>27 May 30. 2006 | Evaluate the following limit $\lim_{t\to3}\frac{\frac{1}{t}-\frac{1}{3}}{t-3}$ |
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## Solution

$$\lim_{t\to3}\frac{\frac{1}{t}-\frac{1}{3}}{t-3}=\frac{\frac{1}{3}-\frac{1}{3}}{3-3}=\frac{0}{0}$$

$$=\lim_{t\to3}\frac{3t\left[\frac{1}{t}-\frac{1}{3}\right]}{3t(t-3)}=\lim_{t\to3}\frac{3-t}{3t(t-3)}=\lim_{t\to3}\frac{-1}{3t}=\frac{-1}{9}$$

# Homework

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| 1 | Find the limit , if it exists | $\lim_{x \to 3} \frac{\sqrt{x+1}-2}{x^2-9}$ | 52 April 9, 2009 A |
| 2 | Evaluate the following limit(if it exists) | $\lim_{x \to 0} \frac{x}{1-\sqrt{1+3x}}$ | 50 November 17, 2008 A |
| 3 | Evaluate the following limit (if it exists) | $\lim_{x \to 1} \frac{\sqrt{5-x}-2}{x-1}$ | 47 November 10.2007 A |
| 4 | Find | $\lim_{x \to 1} \frac{\sqrt{x+3}-2}{x-1}$ | 2 June 12 1990 |
| 5 | Find the limit , if it exists | $\lim_{x \to 1} \frac{\sqrt{x+2}-\sqrt{3}}{x-1}$ | 16 November 2, 1996 |
| 6 | Evaluate the following limit(if it exists) | $\lim_{x \to 0} \frac{\sqrt[3]{x+8}-2}{x}$ | |
| 7 | Evaluate the following limit(if it exists) | $\lim_{x \to 5} \frac{\sqrt{x-1}-2}{x-5}$ | 24 August 3, 2002 |
| 8 | Evaluate the following limit(if it exists) | $\lim_{x \to 0} \frac{1}{x}\left(\frac{1}{x+5}-\frac{1}{5}\right)$ | 31 June 5, 2008 |
| 9 | Find the limit , if it exists | $\lim_{x \to 2} \frac{x^2-4x+4}{x^2+x-6}$ | 26 June 7, 2003 |
| 10 | Let $f(x)=\begin{cases} \frac{\lvert x^2-4\rvert}{x-2} & \text{if } x<2 \\ x+2 & \text{if } x \geq 2 \end{cases}$ find $\lim_{x \to 2} f(x)$ if it exists | | 10 October 27, 1994 |

# Homework

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| 11 | Evaluate each of the following limits, if it exists: | $\lim_{x\to 2}\frac{\sqrt{4-x}-\sqrt{x}}{x-2}$ | 56 July 10, 2010 |
| 12 | Evaluate the following limits, if they exist: | $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{x^2+x}\right)$ | 38 January 15, 2011 |
| 13 | Find the limit , if it exists | $\lim_{x\to 1}\frac{\sqrt{2x^3-1}-1}{x-1}$ | 45 March 28, 2007 |
| 14 | Find the following limit , if it exists | $\lim_{x\to 1}\frac{\sqrt{x+1}-\sqrt{2x}}{x^2-x}$ | 21 March 26, 1998 |

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| 13 | Find the limit , if it exists $\lim_{x\to 1}\frac{\sqrt{2x^3-1}-1}{x-1}$ |

## Solution

$$\lim_{x\to 1}\frac{\sqrt{2x^3-1}-1}{x-1}=\frac{1-1}{1-1}=\frac{0}{0}$$

$$\lim_{x\to 1}\frac{\sqrt{2x^3-1}-1}{x-1}=\lim_{x\to 1}\frac{\left(\sqrt{2x^3-1}-1\right)\left(\sqrt{2x^3-1}+1\right)}{(x-1)\left(\sqrt{2x^3-1}+1\right)}$$

$$=\lim_{x\to 1}\frac{(2x^3-1)-1}{(x-1)\left(\sqrt{2x^3-1}+1\right)}=\lim_{x\to 1}\frac{(2x^3-2)}{(x-1)\left(\sqrt{2x^3-1}+1\right)}=\lim_{x\to 1}\frac{2(x^3-1)}{(x-1)\left(\sqrt{2x^3-1}+1\right)}$$

$$=\lim_{x\to 1}\frac{2(x-1)(x^2+x+1)}{(x-1)\left(\sqrt{2x^3-1}+1\right)}=\lim_{x\to 1}\frac{2(x^2+x+1)}{\left(\sqrt{2x^3-1}-1\right)}=\frac{2(1+1+1)}{1+1}=3$$

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| 14 | Find the following limit , if it exists $\lim_{x\to 1}\frac{\sqrt{x+1}-\sqrt{2x}}{x^2-x}$ |

## Solution

$$\lim_{x\to 1}\frac{\sqrt{x+1}-\sqrt{2x}}{x^2-x}=\frac{\sqrt{2}-\sqrt{2}}{1-1}=\frac{0}{0}$$

$$\lim_{x\to 1}\frac{\sqrt{x+1}-\sqrt{2x}}{x^2-x}=\lim_{x\to 1}\frac{\left(\sqrt{x+1}-\sqrt{2x}\right)\left(\sqrt{x+1}+\sqrt{2x}\right)}{x(x-1)\left(\sqrt{x+1}+\sqrt{2x}\right)}=\lim_{x\to 1}\frac{(x+1)-2x}{x(x-1)\left(\sqrt{x+1}+\sqrt{2x}\right)}$$

$$=\lim_{x\to 1}\frac{1-x}{x(x-1)\left(\sqrt{x+1}+\sqrt{2x}\right)}=\lim_{x\to 1}\frac{-1}{x\left(\sqrt{x+1}+\sqrt{2x}\right)}=\frac{-1}{\sqrt{2}+\sqrt{2}}=\frac{-1}{2\sqrt{2}}$$